

Shear viscosity of neutron-rich nucleonic matter near its liquid-gas phase transition

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Within a relaxation time approach using free nucleon-nucleon cross sections modified by the in-medium nucleon masses that are determined from an isospin- and momentum-dependent effective nucleon-nucleon interaction, we investigate the specific shear viscosity (η/s) of neutron-rich nucleonic matter near its liquid-gas phase transition. It is found that as the nucleonic matter is heated at fixed pressure or compressed at fixed temperature, its specific shear viscosity shows a valley shape in the temperature or density dependence, with the minimum located at the boundary of the phase transition. Moreover, the value of η/s drops suddenly at the first-order liquid-gas phase transition temperature, reaching as low as $4 \sim 5$ times the KSS bound of $\hbar/4\pi$. However, it varies smoothly for the second-order liquid-gas phase transition. Effects of the isospin degree of freedom and the nuclear symmetry energy on the value of η/s are also discussed.

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Transport properties of hot nuclear matter at various densities, such as the shear viscosity, can be extracted from model analyses of heavy-ion collisions. In relativistic heavy-ion collisions, detailed studies have shown that the produced Quark-Gluon-Plasma (QGP) has a very small shear viscosity and behaves almost like an ideal fluid [1, 2]. Specifically, it has been found [3, 4] that the specific shear viscosity, i.e., the ratio of the shear viscosity to the entropy density, of the QGP is only a few times the KSS lower bound of $\hbar/4\pi$ derived from the AdS/CFT correspondence [5]. Also, the specific shear viscosity shows a minimum value around the critical temperature of the hadron-quark phase transition [6, 7]. It is argued in Ref. [6] that the existence of a minimum in the specific shear viscosity is due to the difficulty for the momentum transport in the QGP as its temperature is close to the critical temperature.

The shear viscosity of nucleonic matter is important for understanding various phenomena, such as signatures of the possible liquid-gas phase transition, in heavy-ion collisions at intermediate energies [8, 9]. Because of the short-range repulsive and intermediate-range attractive nature of the nucleon-nucleon interaction, hot nucleonic matter is expected to undergo a liquid-gas phase transition, see, e.g., Refs. [10, 11]. Imprints of such a phase transition on experimental observables, such as the rank distribution of fragments [12], are expected in the multifragmentation process of heavy-ion collisions at intermediate energies [13]. However, while extensive studies have been made to investigate both experimentally and theo-

retically the signatures and nature of the liquid-gas phase transition using various approaches and observables over the last thirty years, see, e.g., Refs. [14–16] for recent reviews, many interesting issues remain to be addressed. In fact, over the last decade much work has been done to better understand the mechanism and nature of the liquid-gas phase transition in isospin asymmetric nucleonic matter, see, e.g., Refs. [17, 18]. In particular, what is the role of the isospin degree of freedom in nuclear thermodynamics? What is the order of the liquid-gas phase transition in neutron-rich nucleonic matter? What are the effects of the density dependence of nuclear symmetry energy on the boundaries of mechanical and chemical instabilities as well as the liquid-gas coexistence line in neutron-rich matter? Answers to these questions are important for understanding both astrophysical observations of supernova explosions and terrestrial experiments done at rare isotope beam facilities. However, many current answers are still under debate. For instance, most models predict that while the liquid-gas phase transition is of first order in isospin symmetric matter, it becomes a continuous transition in isospin asymmetric matter examined at a constant proton fraction. On the other hand, it has been shown that the liquid-gas phase transition is actually still of first order even in isospin asymmetric matter except at the two ending points because of the existence of a spinodal region [19].

Similar to its behavior at the hadron-quark phase transition, the specific shear viscosity of nucleonic matter also shows a minimum value at the vicinity of its liquid-gas phase transition [20–22]. Also, it was speculated that the behavior of the specific shear viscosity at the phase transition may depend on the order of the transition [23]. Thus, further studies on the specific shear viscosity near

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the liquid-gas phase transition may help shed new light on the nature of this transition in neutron-rich matter. Indeed, it has been shown that the boundaries of both mechanical and chemical instabilities responsible for the phase separation [24, 25] and the phase coexistence line [26, 27] in asymmetric nucleonic matter depend on the value of the nuclear symmetry energy $E_{\text{sym}}(\rho)$ at subsaturation densities. It is, however, not known how the $E_{\text{sym}}(\rho)$ affects the specific shear viscosity of nucleonic matter at the liquid-gas phase transition. Since the nuclear matter can undergo the liquid-gas phase transition at different temperatures and densities in intermediate-energy heavy-ion collisions, it is of interest to know how the specific shear viscosity would behave under these various conditions. For example, is the valley shape structure in the temperature and density dependence of the specific shear viscosity of nucleonic matter the result of the liquid-gas phase transition?

In the present study, we use a relaxation time approach to study the specific shear viscosity of neutron-rich nucleonic matter near the liquid-gas phase transition based on a consistent Gibbs construction. We find that the behavior of the specific shear viscosity at the liquid-gas phase transition depends on its order, and that the phase transition can cause a valley structure in the temperature or density dependence of the specific shear viscosity, although it does not necessarily require the existence of a phase transition.

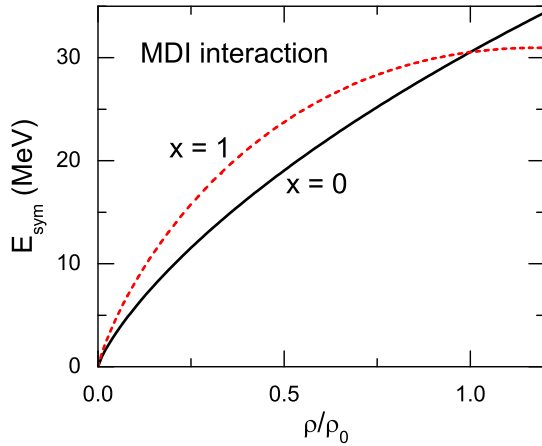


FIG. 1: (Color online) Density dependence of a stiffer ($x = 0$) and a softer ($x = 1$) symmetry energy from the MDI interaction.

For the nucleon-nucleon interaction, we use the isospin- and momentum-dependent interaction proposed in Refs. [28, 29] (hereafter 'MDI') with its parameters fitted to the binding energy -16 MeV and incompressibility 212 MeV of normal nuclear matter at the saturation density $\rho_0 = 0.16 \text{ fm}^{-3}$. For the density dependence of the symmetry energy, the parameter x is used to change its slope parameter $L = 3\rho_0 (dE_{\text{sym}}/d\rho)_{\rho=\rho_0}$ but keeping

its value at saturation density fixed to $E_{\text{sym}}(\rho_0) = 31.6$ MeV. In particular, a stiffer and a softer symmetry energy with $L \approx 60$ MeV and $L \approx 15$ MeV are obtained with $x = 0$ and $x = 1$, respectively, as shown in Fig. 1, corresponding to current uncertainties in the density dependence of the symmetry energy at subsaturation densities [30].

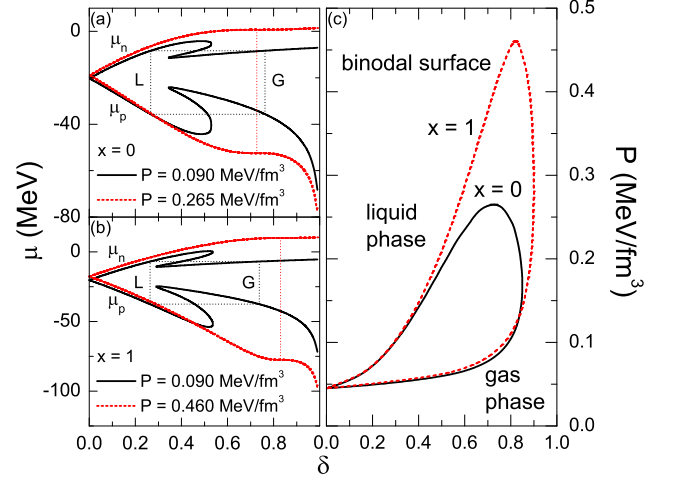


FIG. 2: (Color online) Chemical potential isobar as a function of isospin asymmetry for the stiffer ($x = 0$) (a) and the softer ($x = 1$) symmetry energies (b) and binodal surface (c) for both values of x at temperature $T = 10$ MeV.

To construct the liquid-gas phase transition region in the nuclear phase diagram, we use the Gibbs conditions, i.e., the liquid and gas phases can coexist when they have the same chemical potential ($\mu_l^{(n,p)} = \mu_g^{(n,p)}$), pressure ($P_l = P_g$), and temperature ($T_l = T_g$). Specifically, we plot the chemical potential isobar as a function of the isospin asymmetry δ , defined as $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, for neutrons as well as protons at a certain temperature, and draw a rectangle within the proton and neutron chemical potential isobars. The two ends of the rectangle then correspond to the two coexisting phases, as shown in panels (a) and (b) of Fig. 2 for the stiffer ($x = 0$) and the softer ($x = 1$) symmetry energies, respectively, with the left end point having a smaller isospin asymmetry and a larger density corresponding to a liquid phase (L) and the right end point having a larger isospin asymmetry and a smaller density corresponding to a gas phase (G). This procedure is repeated until the pressure is too low to allow a rectangle to be drawn or too high for the hot nucleonic matter to remain in the chemical instability region, i.e., the chemical potential of neutrons (protons) increases (decreases) monotonically with increasing isospin asymmetry. The coexisting phases at different values of pressure form the binodal surface shown in panel (c) of Fig. 2. The right and the left side of the binodal surface correspond to the gas and the liquid phase, respectively, with the mixed phase inside the binodal surface. The binodal surface thus provides all the information needed

to study the properties of the mixed phase, i.e., the densities and isospin asymmetries of the two coexisting phases as well as their volume fractions. For more details on the liquid-gas phase transition in nucleonic matter, we refer the readers to Refs. [26, 27, 31].

In the phase coexistence region with the liquid phase occupying a volume fraction λ , the average number and entropy densities can be expressed as

$$\rho = \lambda \rho_l + (1 - \lambda) \rho_g, \quad (1)$$

$$s = \lambda s_l + (1 - \lambda) s_g, \quad (2)$$

where $\rho_{l(g)}$ and $s_{l(g)}$ are the number and entropy densities of the liquid (gas) phase, respectively.

For the calculation of the shear viscosity, we consider a stationary flow field in the z direction, i.e., $u_z = f(x)$ in the nucleonic matter where $f(x)$ is an arbitrary function of the coordinate x , and use a similar framework as in Ref. [32]. For a single phase of gas or liquid, the shear force on the particles in a flow layer of a unit area in the $y - z$ plane is equal to the net z -component of momentum transported per sec in the x direction, i.e., the thermal average of the product of the flux $\rho_\tau v_x$ in the x direction and the momentum transfer $p_z - m u_z$ in the z direction [32, 33]

$$F_i = \sum_\tau \langle (p_z - m u_z) \rho_\tau v_x \rangle_i, \quad (3)$$

with $\tau = n$ for neutrons and p for protons, $i = l$ for the liquid phase and g for the gas phase, and m being the nucleon mass. The shear viscosity $\eta_{l(g)}$ is then determined by

$$F_{l(g)} = -\eta_{l(g)} \partial u_z / \partial x \quad (4)$$

for either the liquid phase or the gas phase. We note that the shear viscosity is independent of the flow gradient if $\partial u_z / \partial x$ is sufficiently small.

For a mixed phase of liquid and gas, the matter can be viewed either as gas bubbles in a liquid or liquid droplets in a gas. The matter above and below any flow layer are then either both liquids or both gas unless the flow layer is tangent to the surface of a gas bubble or a liquid droplet, which would have the liquid and the gas on the opposite sides of the flow layer. Since the chance for the latter to happen is infinitesimally smaller for an infinitely large system with liquid droplets and gas bubbles randomly distributed as assumed in the present work, the fraction of the area for particle transport across a flow layer in the liquid is thus λ and that in the gas is $1 - \lambda$, leading to an average shear force on a unit area of flow layer in the mixed phase given by the sum of the contributions from individual phases, i.e.,

$$F = \lambda F_l + (1 - \lambda) F_g = -\eta \partial u_z / \partial x. \quad (5)$$

The average shear viscosity of the mixed phase can then be expressed in terms of those in the liquid or the gas phase as

$$\eta = \lambda \eta_l + (1 - \lambda) \eta_g. \quad (6)$$

Because the density is uniform in each phase, η_l and η_g can be separately calculated using the relaxation time approach as in Ref. [32] based on free nucleon-nucleon cross sections [34] modified by the in-medium nucleon masses [35].

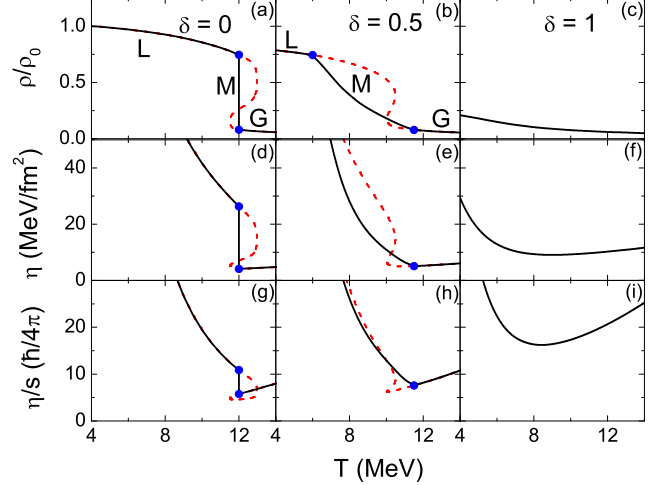


FIG. 3: (Color online) Temperature dependence of the average reduced number density (first row), the shear viscosity (second row), and the specific shear viscosity (third row) at the fixed pressure of $P = 0.1 \text{ MeV/fm}^3$ for isospin symmetric matter ($\delta = 0$) (left column), neutron-rich matter ($\delta = 0.5$) (middle column), and pure neutron matter ($\delta = 1$) (right column) with the stiffer symmetry energy $x = 0$. Solid lines are results including the liquid-gas phase transition with 'L', 'M', and 'G' representing the liquid phase, the mixed phase, and the gas phase, respectively. Dashed lines are results obtained by assuming the liquid-gas phase transition does not happen inside the binodal surface.

Figure 3 displays the temperature dependence of the average reduced number density, the shear viscosity, and the specific shear viscosity, obtained with the stiffer symmetry energy $x = 0$, when the nucleonic matter is heated at the fixed pressure of $P = 0.1 \text{ MeV/fm}^3$. As the temperature increases, the hot nucleonic matter undergoes a phase transition from the liquid phase at lower temperatures to the gas phase at higher temperatures if it has an isospin asymmetry $\delta = 0$ or $\delta = 0.5$ but has no phase transition if the isospin asymmetry is $\delta = 1$. The liquid-gas phase transition is of first order in symmetric nucleonic matter ($\delta = 0$) as shown in Fig. 26 of Ref. [27] by the sudden jump in the entropy per nucleon from the liquid phase to the gas phase as well as the discontinuity of the specific heat at the critical temperature. This leads to the sudden changes in all the thermodynamical quantities and the specific shear viscosity, while the latter evolves smoothly during the phase transition when it changes to a second-order one in neutron-rich matter

($\delta = 0.5$), confirming the expectation of Ref. [23]. Also, the liquid phase has a higher density and a lower temperature than the gas phase as shown in the first row of Fig. 3, leading to a stronger Pauli blocking effect in the liquid phase than in the gas phase. As a result, the liquid phase generally has a larger shear viscosity than the gas phase. For each phase, there are competing density and temperature effects on the evolution of the shear viscosity. As discussed in Ref. [32], an increase in temperature results in more frequent nucleon-nucleon scatterings and weaker Pauli blocking effects, thus reducing the shear viscosity. On the other hand, the nucleon-nucleon scattering cross section decreases with increasing center-of-mass energy of two colliding nucleons as shown in Fig. 2 of Ref. [32], which makes the shear viscosity to increase with increasing temperature especially at very low densities. At higher densities, although the stronger Pauli blocking effect increases the shear viscosity, the smaller in-medium nucleon mass leads to a larger flux between flow layers and a larger relative velocity between two colliding nucleons, thus reducing the shear viscosity. Due to the combination of these effects together with the behavior of the entropy density with respect to temperature and density, the specific shear viscosity decreases in the liquid phase but increases in the gas phase with increasing temperature. The minimum of the specific shear viscosity is exactly located at the critical temperature when a first-order phase transition happens, while it is located at the boundary of the gas phase if the phase transition is of second order. Even for a pure neutron matter without a liquid-gas phase transition, the specific shear viscosity still shows a valley shape in its temperature dependence as a result of the competing effects discussed above.

The liquid-gas phase transition can also happen if the hot nucleonic matter is compressed at a fixed temperature. The density dependence of the pressure, the shear viscosity, and the specific shear viscosity in this case are shown in Fig. 4, again using the stiffer symmetry energy $x = 0$. For the symmetric nuclear matter ($\delta = 0$) that has a first-order liquid-gas phase transition, the pressure remains a constant when it is compressed from the low-density gas phase to the high-density liquid phase. As the nucleonic matter becomes neutron-rich ($\delta = 0.5$) with the phase transition changing to a second-order one, the pressure continues to increase with increasing density in the mixed phase. For the pure neutron matter, it again does not show a liquid-gas phase transition when it is compressed at a fixed temperature. It is shown in the second row of Fig. 4 that the occurrence of the mixed phase in the hot nucleonic matter when it is compressed at a constant temperature generally increases the value of the shear viscosity compared with the case by assuming that the liquid-gas phase transition does not happen. Also, the specific shear viscosity always has a minimum value, and we found that this is due to the difference in the increase of the shear viscosity and the entropy density with increasing density, even for the case of pure neutron matter without a liquid-gas phase transition. Interest-

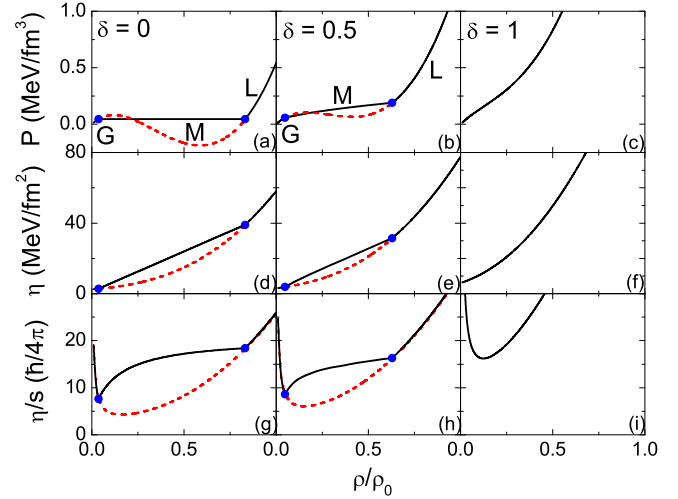


FIG. 4: (Color online) Density dependence of the pressure (first row), the shear viscosity (second row), and the specific shear viscosity (third row) at temperature $T = 10$ MeV for isospin symmetric matter ($\delta = 0$) (left column), neutron-rich matter ($\delta = 0.5$) (middle column), and pure neutron matter ($\delta = 1$) (right column) with the stiffer symmetry energy $x = 0$. Solid lines are results including the liquid-gas phase transition with 'L', 'M', and 'G' representing the liquid phase, the mixed phase, and the gas phase, respectively. Dashed lines are results obtained by assuming the liquid-gas phase transition does not happen inside the binodal surface.

ingly, the density at which the specific shear viscosity has a minimum value is again located at the boundary of the gas phase for $\delta = 0$ and $\delta = 0.5$ independent of the phase transition order.

Since different values of pressure and temperature are reached in intermediate-energy heavy-ion collisions, it is of interest to study the specific shear viscosity of nucleonic matter at the liquid-gas phase transition under different conditions. In the upper panels of Fig. 5, we compare the temperature dependence of the specific shear viscosity at different pressures for isospin symmetric ($\delta = 0$) and neutron-rich ($\delta = 0.5$) nucleonic matter as well as pure neutron matter ($\delta = 1$) with both the stiffer ($x = 0$) and the softer ($x = 1$) symmetry energies. It is seen that the temperature at which the specific shear viscosity has a minimum increases with increasing value of the fixed pressure, similar to the results in Refs. [6, 23]. Also, for larger fixed pressures the minimum value of the specific shear viscosity is smaller for $\delta = 0$ and $\delta = 0.5$ but seems to be independent of the pressure for $\delta = 1$. In the lower panels of Fig. 5, we display the density dependence of the specific shear viscosity for different temperatures. Similarly, the density at which the specific shear viscosity has a minimum value increases with increasing value of the fixed temperature, and the minimum value is smaller at higher

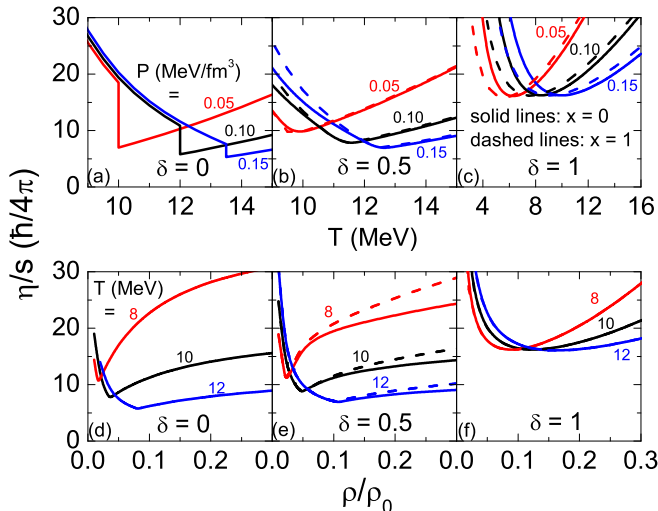


FIG. 5: (Color online) Temperature (upper panels) and density (lower panels) dependence of the specific shear viscosity at different fixed pressures and temperatures, respectively, in isospin symmetric matter ($\delta = 0$), neutron-rich matter ($\delta = 0.5$), and pure neutron matter ($\delta = 1$) for both symmetry energies $x = 0$ and $x = 1$.

fixed temperatures for $\delta = 0$ and $\delta = 0.5$ but is insensitive to the temperature for $\delta = 1$. It is worthwhile to note that with further increase in pressure or temperature, the minimum value of the specific shear viscosity decreases and then levels off until the pressure or the temperature is too high for the nucleonic matter to have a liquid-gas phase transition. The resulting lower limit of the specific shear viscosity of nucleonic matter is about $4 \sim 5 \hbar/4\pi$ for isospin symmetric nucleonic matter and is generally smaller than that in neutron-rich nucleonic matter as discussed in Ref. [32]. As seen in panel (c) of Fig. 2, the stiffness of the symmetry energy only slightly affects the gas side of the phase boundary, thus having only negligible effects on the location of the minimum value of the specific shear viscosity. However, due to the difference in the phase coexistence region for the stiffer ($x = 0$) and the softer ($x = 1$) symmetry energy, different specific shear viscosities are obtained in the mixed-phase region, with the softer symmetry energy ($x = 1$) giving a larger value compared with the stiffer symmetry energy ($x = 0$) as shown in panels (b) and (e) of Fig. 5. For the

pure neutron matter without the liquid-gas phase transition under a fixed pressure, it looks like that the specific shear viscosity for $x = 1$ is similar to that for $x = 0$ obtained under a slightly smaller fixed pressure. On the other hand, the specific shear viscosities from different symmetry energies are the same for pure neutron matter compressed at a fixed temperature as indicated in panel (f) of Fig. 5.

To summarize, using the relaxation time approach, we have studied the specific shear viscosity of neutron-rich nucleonic matter near its liquid-gas phase transition boundary constructed from the Gibbs conditions. A valley shape is observed in the temperature or density dependence of the specific shear viscosity even in the absence of the phase transition. The value of the specific shear viscosity suddenly drops at the first-order liquid-gas phase transition temperature, while it varies smoothly for the second-order phase transition. Moreover, the density dependence of the symmetry energy is found to affect the value of the specific shear viscosity of nucleonic matter in the mixed-phase region, although it has little effects on the location of its minimum. Our results are expected to be useful for investigating the nature and signatures of the liquid-gas phase transition in neutron-rich matter using intermediate-energy heavy-ion collisions induced by rare isotopes.

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